Submission to

IWTO Technical Committee Working Group:

RAW WOOL GROUP

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Mathematical Combination of Measured Fibre Diameter Distribution Histograms for Greasy and Scoured Wool

by

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Summary

IWTO-31-86(E) includes a formula for combining Mean Fibre Diameter for sublots where the mean diameter is measured using Airflow. IWTO-12-93(E) includes formulae for combining Mean Fibre Diameter and Coefficient of Variation, where the diameter distribution data are also available.

The development and commercialisation of rapid techniques for measuring fibre diameter distributions for lots of greasy and scoured wool has generated increasing interest in obtaining these data for consignments of the same. While these data can be obtained by measuring a single sample drawn from the consignment, in practice it is more common for sublots that make up the consignment to be sampled and measured and the results of these individual tests combined, using the formulae in IWTO-12-93(E). However IWTO-12-93(E) does not provide a formula for producing a combined histogram from the individual sublot histograms and there is increasing demand for this to be also available.

Consequently a mathematical formula has been derived for estimating the fibre diameter distribution histogram for a blend of wool fibres, from the distribution histograms for each component of the blend. The derivation of this formula is described in this submission. The submission recommends that IWTO-12-93(E) be amended to include this formula and details a number of other, associated amendments.

IWTO Combination Formulae

The mathematical formula for calculating the mean fibre diameter of a blend of wool fibres, where distribution data is available for each component of the blend, was originally derived by Fell *et al*[1], utilising notation and relations developed by Palmer[2] and later extended by Montfort[3]. The mean fibre diameter so derived, was expressed as $(l,d)_m$, where $(l,d)_m$ represents the arithmetic mean diameter (d) when it is distributed proportionally to length (l).

This work was extended by David *et al*[4] who derived a mathematical formula for estimating the population variance of a blend of wool fibres, where the distribution data is available for each component of the blend. This resulted in a relationship which provided an estimate of the coefficient of variation of fibre diameter for the blend.

These formulae are currently incorporated in IWTO-12-93(E). The notation developed by Palmer has been simplified, because for equations of such complexity his notation is extremely cumbersome. However it must be noted that the mean diameter is always a length-biased estimate.

The formulae currently incorporated in IWTO-12-93(E) , and the definitions of the notation used, are reproduced below.

Notation

(a) For the ***i***th lot :

$\mathbf{B}_i$	=	wool base percent
$\mathbf{M}_i$	=	nett mass of greasy or scoured wool in kilograms
$\mathbf{D}_i$	=	mean fibre diameter in micrometres
$\mathbf{C}_i$	=	fractional coefficient of variation

where $\mathbf{C}_i = \dfrac{\mathbf{S}_i}{\mathbf{D}_i}$

$\mathbf{S}_i$ = estimated standard deviation of diameter in micrometres

where i = 1,2,3......***k*** and ***k*** is the total number of lots in the consignment.

(b) For the consignment :

D	=	mean fibre diameter in micrometres
C	=	fractional coefficient of variation
S	=	estimated standard deviation of diameter in micrometres

$$\mathbf{D} = \frac{\sum_i \left[\mathbf{B}_i{*}\mathbf{M}_i / \left(\mathbf{D}_i(1 + \mathbf{C}_i^2)\right)\right]}{\sum_i \left[\mathbf{B}_i{*}\mathbf{M}_i / \left(\mathbf{D}_i^2{*}(1 + \mathbf{C}_i^2)\right)\right]} \quad (1)$$

$$\mathbf{C} = \left\{ \frac{\sum_i \mathbf{B}_i{*}\mathbf{M}_i}{\mathbf{D}^2{*}\sum_i \left[\mathbf{B}_i{*}\mathbf{M}_i / \left(\mathbf{D}_i^2{*}(1 + \mathbf{C}_i^2)\right)\right]} - 1 \right\}^{1/2} \quad (2)$$

Combining Fibre Diameter Distribution Histograms

The derivation of the equation for calculating an estimate of the fibre diameter distribution histogram for a blend of wool fibres, from the measured distribution histograms of each component is detailed in Appendix B. The equation so derived is presented below (equation 3).

Fibre diameter distribution histograms are generally produced by measuring the diameter of a specified number of individual fibre snippets. The individual fibre measurements are grouped into frequency bins or class intervals representing a range of one micrometre. The number of bins provided by instrumental techniques such as FFDA, LASERSCAN and OFDA is commonly limited to 80, over a range of 0 to 80 micrometres, with a class interval of 1 micrometre, but this is not an absolute restriction, and is chosen simply because it generally encompasses the range of diameter normally found for wool fibres. For Projection Microscope measurements the range is restricted by the physical characteristics of the measurement scale provided with the instrument and the optics of the instrument, but nevertheless the range can readily extend above 80 micrometres.

In deriving the equation for combining histogram data it has been assumed that 80 bins of 1 micrometre range are used by the measurement system. However this does not limit the application of the equation and it can be readily applied to measured distributions based on a different number of bins with a different class interval..

Notation

(a) For the i^{th} lot :

	B_i	=	wool base percent;
	M_i	=	nett mass of greasy or scoured wool in kilograms;
	D_i	=	mean fibre diameter in micrometres;
	C_i	=	fractional coefficient of variation
where	C_i	=	$\frac{S_i}{D_i}$;
	S_i	=	estimated standard deviation of diameter in micrometres;
	$n_{i,k}$	=	total number of fibre snippets measured for the k^{th} bin of i^{th} component;
	d_k	=	the nominal diameter of the k^{th} bin of the measured diameter distribution; and
where	i	=	1,2,3......m and m is the total number of lots in the consignment.;
and	k	=	0,1,2....80.

(b) For the consignment :

n_k	=	the calculated number of snippets in the k^{th} class interval for a randomly selected specimen of the consignment.

From Appendix B, equation B10

$$n_k = \left[\Sigma_i\left(\frac{B_i * M_i * n_{i,k}}{\Sigma_k d_k^2 * n_{i,k}}\right)\right] * \left[\frac{\Sigma_k \Sigma_i n_{i,k}}{\Sigma_k \Sigma_i \left(B_i * M_i * n_{i,k} / \left(\Sigma_k d_k^2 * n_{i,k}\right)\right)}\right] \quad (3)$$

Equation 3 consists of two parts. The expression included in the first summation i.e

$$\Sigma_i\left(\frac{\mathbf{B}_i*\mathbf{M}_i*\mathbf{n}_{i,k}}{\Sigma_k \mathbf{d}_k^2*\mathbf{n}_{i,k}}\right)$$

is the calculated fibre diameter distribution for the blend. However this expression on its own, while predicting a distribution which reflects the distribution of fibres in the blend in percentage terms, results in a total number of fibres that is less than the total number that were actually measured. The second half of the equation is an expression that normalises the calculated distribution so that the total number of fibres in the calculated distribution is the same as the total number measured for the components of the blend. The normalisation process is described in Appendix B

The other feature of the equation is that it will result in fractional numbers of fibres in each of the bins. This is an artefact of the mathematics. However if the calculated distribution data is used to calculate the mean fibre diameter and the coefficient of variation for the blend then the raw numbers predicted by this equation, rather than rounded integers should be used. If this is not done then rounding errors in the mean and coefficient of variation may occur. In any event rounding to whole numbers may introduce rounding errors which result in a total number of fibres which is less than or greater than the total number that were originally measured.

The transformation of the frequency distribution into a percentage frequency distribution avoids the presentation of fractional numbers of snippets in the class intervals. This also produces distributions that can be readily compared for different consignments.

It is recommended that calculations using this equation should utilise floating point numbers if possible to prevent rounding errors. Alternatively calculations should utilise 12 decimal places.

If the distribution derived from equation 3 is actually used to calculate the mean fibre diameter for the blend then this accurately reproduces the value calculated from equation 1. However for the coefficient of variation, small differences have been observed from the values calculated from equation 2. These differences are usually in the third decimal place and are generally not observed if the values are rounded to one decimal place. The reason for this difference has not yet been identified but is believed to be associated with the assumptions implicit in the derivation of equations 1 and 2 in relation to the shape of the fibre diameter distributions. Palmer[2] assumed a log normal distribution and David *et al*[4] assumed a normal distribution. The derivation of equation 3 makes no such assumptions. Nevertheless for commercial purposes the differences are insignificant and can therefore be ignored.

Figures 1 and 2 are fibre diameter distributions measured by LASERSCAN for two lots of greasy wool (Component A and Component B). Figures 3, 4 and 5 show the fibre diameter distribution of a blend of these lots, calculated from equation 3, when the Wool Base is held constant and the nett weights varied in the ratio A:B = 3:1, 1:1, 1:3.

Figure 1

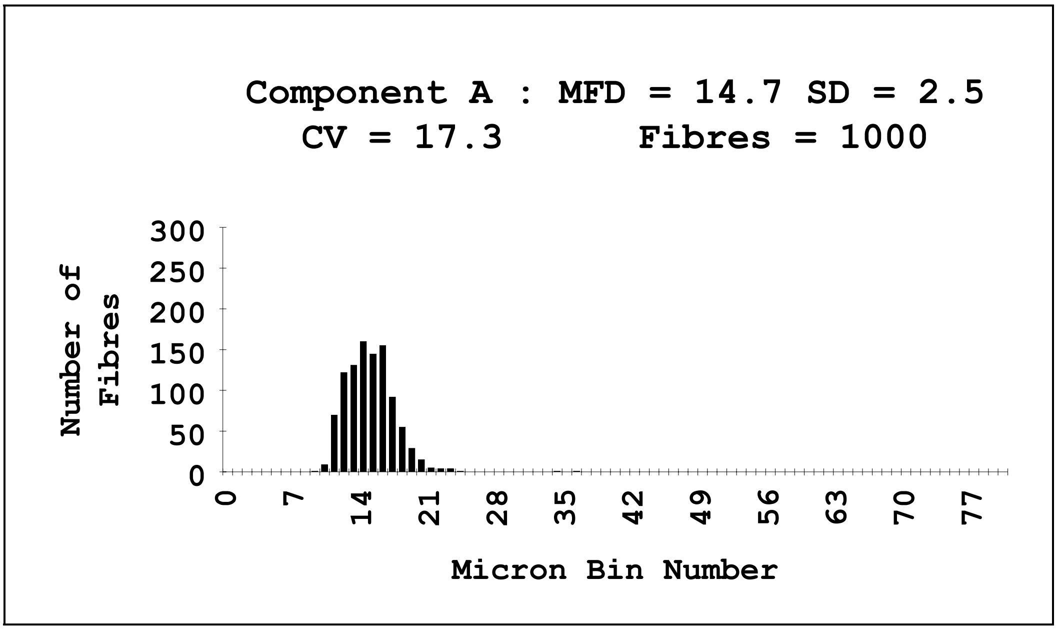
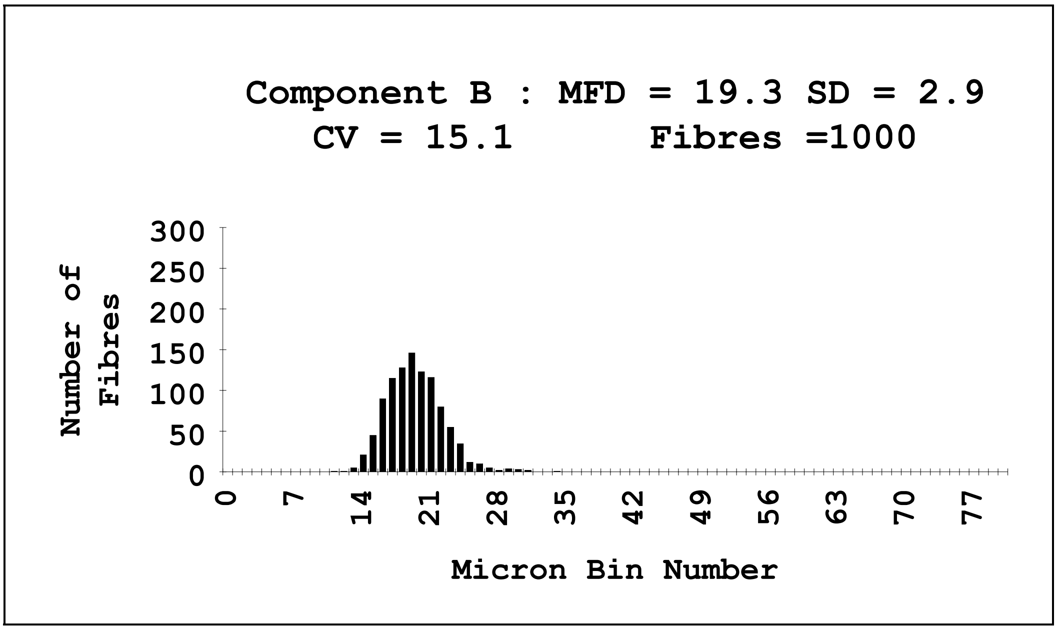


Figure 2



The values for Mean Fibre Diameter (MFD), Standard Deviation (SD) and Coefficient of Variation (CV) that are displayed in Figure 3, 4 and 5 have been calculated from the distributions and then rounded to one decimal place. In Table 1 the values calculated from equations 1 and 2, are compared with the values calculated from the distributions produced from equation 3, but the data has been rounded to 3 decimal places. The differences that occur are small and as previously stated, are confined to the third decimal place. This comparison is provided as an illustration, but the equation has been extensively tested using blends with a wide and with a narrow range of mean fibre diameter in the individual components.

Table 1

Ratio A:B	MFD		SD		CV	
	Eq 3	Eq 1	Eq 3	Eq 1 &2	Eq 3	Eq 2
3:1	15.468	15.468	3.117	3.117	20.149	20.155
1:1	16.419	16.419	3.496	3.496	21.290	21.292
1:3	17.689	17.689	3.572	3.574	20.228	20.239

Figure 3

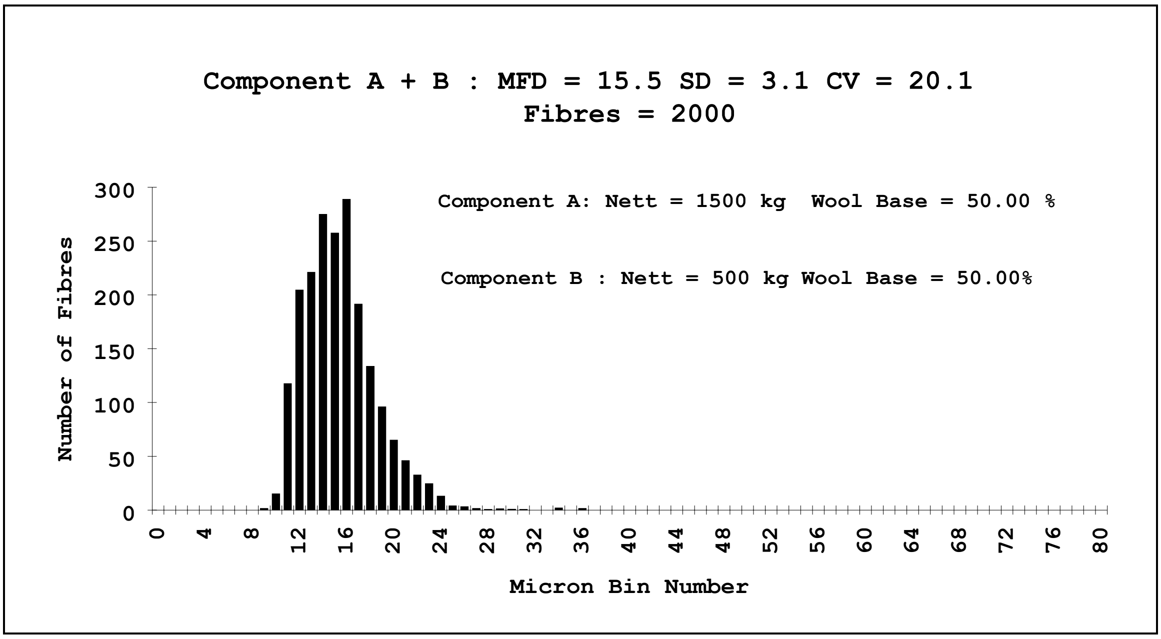


Figure 4

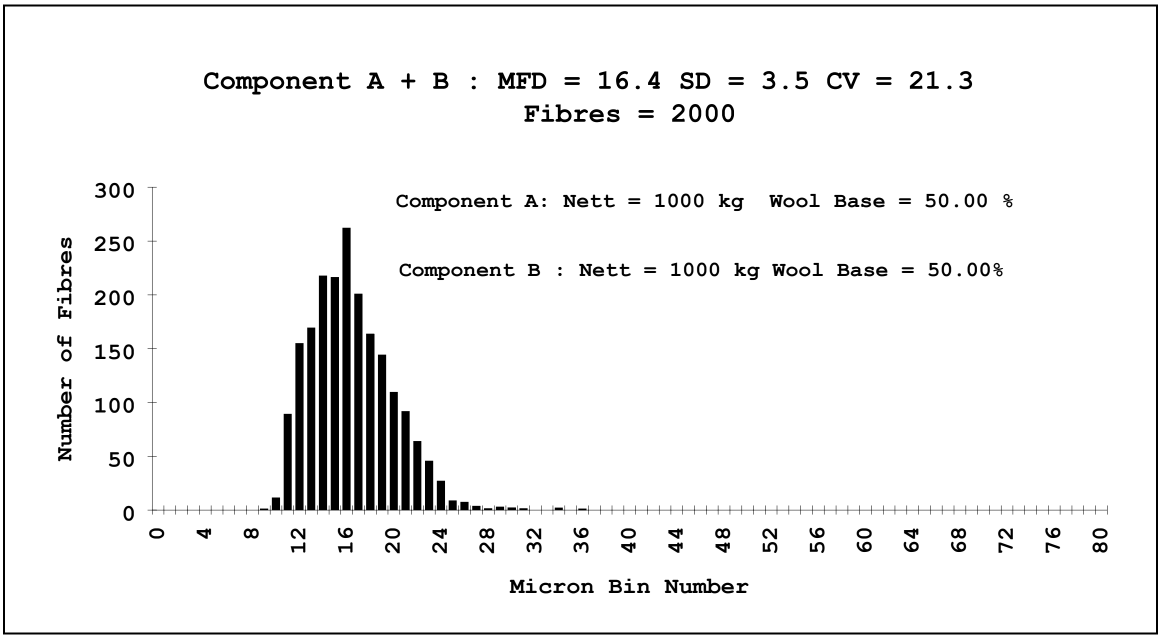
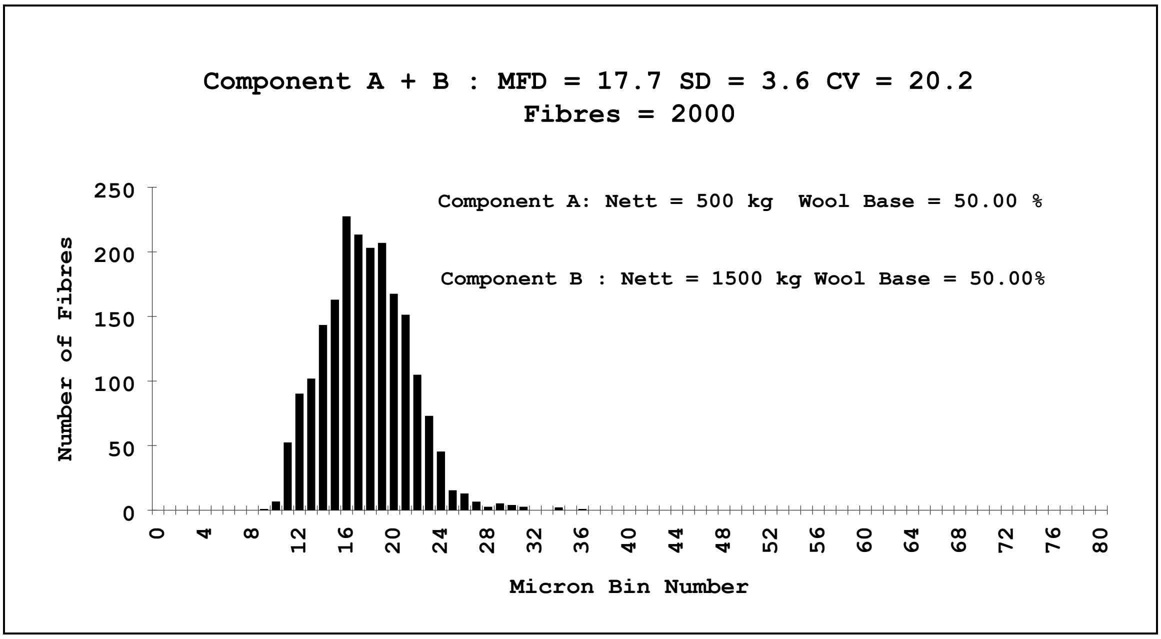


Figure 5



Recommendations

It is recommended that :

1. IWTO-12-93(E) be amended as specified in Appendix A to incorporate equation 3

2. IWTO-31-86(E) is amended to incorporate equations 1, 2 and 3.

Bibliography

[1] K. T. Fell, M. W. Andrews, and J. F. P. James. *J. Text . Inst.*, 1972, **63**, 125.
[2] R. C Palmer. *Proc. IWTO Tech Cttee*, 1947, **June**, 39.
[3] F. Montfort. "Aspects Scientifiques de l'Industrie Lainière", Dunod, Paris, 1960.
[4] H. G. David and M. W. Andrews. *J. Text. Inst.*, 1972, **63**, 637

APPENDIX A

The following is a draft amendment to IWTO-12-93(E). Amendments to the text are highlighted in bold italics. Where this inappropriate the relevant amendment is enclosed within a boxed outline.and the notation "[amendment]" placed at the bottom right hand of this outline.

7. EXPRESSION OF RESULTS

7.1 Report on Individual Tests

The following information shall be reported :

(a) In the case of card sliver or top, the number of lengths taken to form the subsample.

(b) The mean fibre diameter to one decimal place.

(c) The number of test specimens measured

The following information may be reported :

(i) The standard deviation (SD) of fibre diameter in micrometres to one decimal place.

(ii) The coefficient of variation (CV) of fibre diameter as a percentage to the nearest whole number.

(iii) The distribution of fibre diameter as a frequency table ***or as a frequency histogram*** with data grouped into classes of one micrometre size, and integer micrometre values as the midpoints of the class intervals.

NOTE : The mean diameter and the standard deviation of diameter are calculated from the pooled distributions of the measured test specimens.

NOTE : Estimates of the precision of the method are given in Appendix D.

7.2 Combination of Mean Fibre Diameter, Standard Deviation, CV ***and Diameter Distributions***

The combined values of a consignment shall be calculated as set out below. As it is usual to report mean and coefficient of variation of the components of a consignment, the combination has been calculated using these values.

The following notation is used :

(a) For the ith lot :

$\mathbf{B}_i$ = wool base percent
$\mathbf{M}_i$ = nett mass of greasy or scoured wool in kilograms

	$\mathbf{D}_i$	=	mean fibre diameter in micrometres
	$\mathbf{C}_i$	=	fractional coefficient of variation
where	$\mathbf{C}_i$	=	$\frac{\mathbf{S}_i}{\mathbf{D}_i}$
	$\mathbf{S}_i$	=	estimated standard deviation of diameter in micrometres
where	i	=	1,2,3......m and m is the total number of lots in the consignment.

	$\mathbf{n}_{i,k}$	=	the number of fibre snippets measured for the k^{th} bin of i^{th} component
	$\mathbf{d}_k$	=	the nominal diameter of the k^{th} bin of the measured diameter distribution
where	k	=	0,1,2....80

[Amendment]

(b) For the consignment :

D	=	mean fibre diameter in micrometres
C	=	fractional coefficient of variation
S	=	estimated standard deviation of diameter in micrometres.

$\mathbf{n}_k$	=	the calculated number of snippets in the k^{th} bin for the consignment.

[Amendment]

7.2.1 Consignment Mean Fibre Diameter

Due to the differences between Laserscan and the other instruments in estimating the Coefficient of Variation, only data from Laserscan can be combined.

Calculate the mean fibre diameter from the following formula (18) :

$$\mathbf{D} = \frac{\sum_i \left[\mathbf{B}_i * \mathbf{M}_i / \left(\mathbf{D}_i (1 + \mathbf{C}_i^2) \right) \right]}{\sum_i \left[\mathbf{B}_i * \mathbf{M}_i / \left(\mathbf{D}_i^2 * (1 + \mathbf{C}_i^2) \right) \right]}$$

[Amendment]

7.2.2 Consignment Fractional Coefficient of Variation

Calculate the combined fractional coefficient of variation of fibre diameter from the following formula (16) :

$$\mathbf{C} = \left\{ \frac{\sum_i \mathbf{B}_i * \mathbf{M}_i}{\mathbf{D}^2 * \sum_i \left[\mathbf{B}_i * \mathbf{M}_i / \left(\mathbf{D}_i^2 * (1 + \mathbf{C}_i^2) \right) \right]} - 1 \right\}^{1/2}$$

[Amendment]

7.2.3 Consignment Standard Deviation

Calculate the combined fractional coefficient of variation as above. Then calculate the standard deviation for the consignment from the following formula :

$$\mathbf{S} = \mathbf{CD}$$

7.2.4 Consignment Fibre Diameter Distribution Histogram.

Calculate the combined fibre diameter distribution from the distributions of the individual components using the following formula

$$\mathbf{n}_k = \left[\sum_i \left(\frac{\mathbf{B}_i * \mathbf{M}_i * \mathbf{n}_{i,k}}{\sum_k \mathbf{d}_k^2 * \mathbf{n}_{i,k}} \right) \right] * \left[\frac{\sum_k \sum_i \mathbf{n}_{i,k}}{\sum_k \sum_i \left(\mathbf{B}_i * \mathbf{M}_i * \mathbf{n}_{i,k} / \left(\sum_k \mathbf{d}_k^2 * \mathbf{n}_{i,k} \right) \right)} \right]$$

7.2.5 Reporting of Results

The following shall be reported for a consignment :

(a) The individual mean fibre diameter of the components.

(b) The mean fibre diameter of the consignment, in micrometres to one decimal place.

The following may be reported :

(i) Coefficient of variation of the consignment as a percentage to the nearest whole number.

(ii) The standard deviation of the consignment in micrometres to one decimal place.

(iii) The distribution of fibre diameter in the consignment as a frequency table or as a frequency histogram with data grouped into classes of one micrometre size, and integer micrometre values as the midpoints of the class intervals.

(iv) The standard deviation (SD) of fibre diameter in micrometres to one decimal place for each component.

(v) The coefficient of variation (CV) of fibre diameter as a percentage to the nearest whole number for each component

(vi) The distribution of fibre diameter for each component as a frequency table or as a frequency histogram with data grouped into classes of one micrometre size, and integer micrometre values as the midpoints of the class intervals.

APPENDIX B

Derivation of an Equation to Combine Measured Distribution Histograms

The derivation of an equation for calculating an estimate of the fibre diameter distribution histogram for a blend of wool fibres from the measured distribution histograms of each component was based on the following assumptions:

- the distribution histograms for each component are derived from measurement of a specified number of fibre snippets of uniform length;
- the distribution histograms for each component have the same number of class intervals, and the same class interval range;
- this population of fibre snippets is representative of a population of snippet equivalents in the component, assuming that every fibre in the component is divided into snippets of the same length as the sample.; and
- the fibres are circular in cross section and of uniform linear density.

The method for deriving the combination equation was as follows :

- use the diameter distribution data to calculate the clean masses of fibres in each class interval for the component;
- total the clean masses so derived to obtain the total mass of fibre in each class interval in the blend;
- calculate the total number of snippet equivalents in each class interval for the blend; and
- calculate a number distribution for the blend based on the same class intervals.

Notation :

General :

ρ	=	density of wool fibre;
l	=	length of snippets;
i	=	1,2,3,......m where m is the number of components; and
k	=	1,2,3,......80 class intervals of 1 micrometre.

For the i^{th} component :

B_i	=	wool base percent;
M_i	=	nett mass of greasy or scoured wool;
P_i	=	clean mass of wool;
$P_{i,k}$	=	clean mass of wool in the k^{th} class interval;
p_i	=	the clean mass of the measured specimen;
$p_{i,k}$	=	the clean mass of fibre in the k^{th} class interval of the specimen;
N_i	=	total calculated number of snippet equivalents of length l ;
$N_{i,k}$	=	total calculated number of snippet equivalents of length l in the k^{th} class interval; and
n_i	=	number of snippets measured.

For the blend :

N	=	total calculated number of snippet equivalents of length l;
N_k	=	total calculated number of snippet equivalents of length l in the k^{th} class interval;
n	=	total number of snippets measured; and
P_k	=	total clean mass of fibre in the k^{th} class interval.

Calculate the clean mass of fibres in the k^{th} class interval of the i^{th} specimen :

$$p_{i,k} = (\pi/4)*\rho*d_k^2*n_{i,k}*l \qquad \text{(B1)}$$

Calculate the total specimen mass for the i^{th} component by summing the calculated class interval masses :

$$p_i = \Sigma_k p_{i,k} = (\pi/4)*\rho*l*\Sigma_k d_k^2*n_{i,k} \qquad \text{(B2)}$$

Calculate the clean mass of the i^{th} component :

$$P_i = B_i*M_i/100 \qquad \text{(B3)}$$

Calculate the total clean mass of the ***k***th class interval for the ***i***th component :

$$\mathbf{P}_{i,k} = \frac{\mathbf{p}_{i,k}}{\mathbf{p}_i} * \mathbf{P}_i = \frac{\mathbf{B}_i * \mathbf{M}_i}{100} * \frac{\mathbf{d}_k^2 * \mathbf{n}_{i,k}}{\sum_k \mathbf{d}_k^2 * \mathbf{n}_{i,k}} \qquad \text{(B4)}$$

Calculate the total clean mass in the ***k***th class interval for the blend by summing the calculated clean masses in the ***k***th class interval of each component :

$$\mathbf{P}_k = \sum_i \mathbf{P}_{i,k}$$

Using equation B4 :

$$\mathbf{P}_k = \sum_i \left(\frac{\mathbf{B}_i * \mathbf{M}_i}{100} * \frac{\mathbf{d}_k^2 * \mathbf{n}_{i,k}}{\sum_k \mathbf{d}_k^2 * \mathbf{n}_{i,k}} \right)$$

$$\mathbf{P}_k = \frac{\mathbf{d}_k^2}{100} * \sum_i \left(\frac{\mathbf{B}_i * \mathbf{M}_i * \mathbf{n}_{i,k}}{\sum_k \mathbf{d}_k^2 * \mathbf{n}_{i,k}} \right) \qquad \text{(B5)}$$

Calculate the total number of snippet equivalents in the ***k***th class interval for the blend :

$$\mathbf{N}_k = \mathbf{P}_k / ((\pi/4) * \rho * \mathbf{d}_k^2 * l) \qquad \text{(B6)}$$

Calculate the total number of snippet equivalents in the blend :

$$\mathbf{N} = \sum_k \mathbf{N}_k$$

Substituting B6 :

$$\mathbf{N} = \frac{1}{(\pi/4) * \rho * l} * \sum_k \frac{\mathbf{P}_k}{\mathbf{d}_k^2} \qquad \text{(B7)}$$

Calculate the total number of snippets measured for the blend :

$$\mathbf{n} = \sum_i \mathbf{n}_i \qquad \text{(B8)}$$

or

$$\mathbf{n} = \sum_k \sum_i \mathbf{n}_{i,k} \qquad \text{(B8a)}$$

Assume that a random specimen of **n** snippets from the blend will contain numbers of snippets in each class interval in the same proportion as the total numbers of snippet equivalents in each class interval of the blend, and scale down to calculate the raw histogram :

$$\mathbf{n}_k = \mathbf{N}_k * \frac{\mathbf{n}}{\mathbf{N}}$$

Substituting B6 and B7 :

$$\mathbf{n}_k = \frac{\mathbf{P}_k}{(\pi/4)*\rho*\mathbf{d}_k^{2}*l} * \mathbf{n} * \frac{(\pi/4)*\rho*l}{\Sigma_k \mathbf{P}_k/\mathbf{d}_k^{2}}$$

By rearrangement and simplifying :

$$\mathbf{n}_k = \frac{\mathbf{P}_k}{\mathbf{d}_k^{2}} * \frac{\Sigma_i \mathbf{n}_i}{\Sigma_k \mathbf{P}_k/\mathbf{d}_k^{2}}$$

Substituting B5 and rearranging gives :

$$\mathbf{n}_k = \left[\Sigma_i\left(\frac{\mathbf{B}_i*\mathbf{M}_i*\mathbf{n}_{i,k}}{\Sigma_k \mathbf{d}_k^{2}*\mathbf{n}_{i,k}}\right)\right]*\left[\frac{\Sigma_i \mathbf{n}_i}{\Sigma_k\Sigma_i\left(\mathbf{B}_i*\mathbf{M}_i*\mathbf{n}_{i,k}/\left(\Sigma_k \mathbf{d}_k^{2}*\mathbf{n}_{i,k}\right)\right)}\right]$$

(B9)

where $\mathbf{n}_k$ is the estimated number of fibres in the k^{th} class interval of a specimen of the blend with a total number of fibres equivalent to the total number of fibres measured for all the components of the blend. By substituting equation B8a equation B9 can be expressed in a slightly different way to facilitate calculation from the raw histogram data for each component.

$$\mathbf{n}_k = \left[\Sigma_i\left(\frac{\mathbf{B}_i*\mathbf{M}_i*\mathbf{n}_{i,k}}{\Sigma_k \mathbf{d}_k^{2}*\mathbf{n}_{i,k}}\right)\right]*\left[\frac{\Sigma_k\Sigma_i \mathbf{n}_{i,k}}{\Sigma_k\Sigma_i\left(\mathbf{B}_i*\mathbf{M}_i*\mathbf{n}_{i,k}/\left(\Sigma_k \mathbf{d}_k^{2}*\mathbf{n}_{i,k}\right)\right)}\right]$$

(B10)