Submission to

IWTO Technology and Standards Committee:

RAW WOOL GROUP

Cape Town - April 1996

Mathematical Combination by Subtraction of Measured Fibre Diameter Distributions of Greasy and Scoured Wool

by

P J Sommerville[a]

(a) AWTA Ltd, PO Box 190, Guildford, NSW 2161, Australia

SUMMARY

At the IWTO conference in Nice in December 1995 Semmel and Sommerville[1] submitted a paper to the Raw Wool Group which described a new equation that enabled the combination of fibre diameter distributions for individual lots to obtain a distribution for a delivery.

The paper also recommended the IWTO-31-86 should be amended to include the new formula and other formulae for combining data derived from measurement systems which provided distribution information. This recommendation was accepted and a revision of IWTO-31-86 has been submitted to the Raw Wool Group at the Cape Town Conference[2]. As part of this process the formula was further developed to provide a formula for calculation the diameter distribution of the remainder of a delivery after a small portion (<5%)has been removed from the delivery. The derivation of this formula is provided in this paper for future reference. The paper is supplementary to the previous paper, which contains a more detailed description of the process involved and the assumptions made.

DERIVATION

Notation:

General :

ρ = the density of wool fibre;
ℓ = length of snippets
i = 1,2,3....q where q is the number of components; and
k = 1,2,3,.....80 class intervals of 1 micrometre

For the ith component of the delivery :

B_i = Wool Base (%);
M_i = nett mass of greasy or scoured wool (kg);
P_i = clean mass of wool (kg);
$P_{i,k}$ = clean mass of wool in the kth class interval;
p_i = clean mass of the measured specimen;
$p_{i,k}$ = the clean mass of fibre in the kth class interval of the specimen;
N_i = total calculated number of snippet equivalents of length ℓ ; and
n_i = number of snippets measured.

For the remainder of the delivery :

N_r = total calculated number of snippet equivalents of length ℓ;
$N_{r,k}$ = total calculated number of snippet equivalents of length ℓ in the kth class interval;
n_r = number of snippets measured; and
$P_{r,k}$ = clean mass of fibre in the kth class interval.

For the component subtracted from the delivery :

B_s = Wool Base (%); and

[1] P J Semmel and P J Sommerville, "Mathematical Combination of Measured Fibre Diameter Histograms for Greasy and Scoured Wool", IWTO Raw Wool Group, Nice, Dec. 1995

[2] P J Sommerville, "Revision of IWTO-31-86(E) to Incorporate Combination Formulae from other IWTO Test Methods", IWTO Raw Wool Group, Cape Town, April 1996

M_s	=	nett mass of greasy or scoured wool
n_s	=	number of snippets measured;
$n_{s,k}$	=	number of snippets in the kth class interval; and
$P_{s,k}$	=	clean mass of wool in the kth class interval.

Calculate the clean mass of fibres in the kth class interval of the ith specimen :

$$p_{i,k} = \frac{\pi}{4}\rho\ell d_k^2 n_{i,k} \tag{1}$$

Calculate the total specimen mass for the ith component by summing the calculated class interval masses :

$$p_i = \sum_{k=1}^{80} p_{i,k} = \frac{\pi}{4}\rho\ell\sum d_k^2 n_{i,k} \tag{2}$$

Calculate the clean mass of the ith component :

$$P_i = B_i M_i / 100 \tag{3}$$

Calculate the total clean mass of the kth class interval for the ith component :

$$P_{i,k} = \frac{p_{i,k}}{p_i} P_i = \frac{B_i M_i}{100} * \frac{d_k^2 n_{i,k}}{\sum_{k=1}^{80} d_k^2 n_{i,k}} \tag{4}$$

Calculate the total clean mass in the kth class interval for the delivery by summing the calculated clean masses in the kth class interval of each component :

$$P_k = \sum_{k=1}^{80} P_{i,k} \tag{5}$$

Calculate the total clean mass in the kth class interval for the remainder of the delivery :

$$P_{r,k} = \sum P_{i,k} - P_{s,k} \tag{6}$$

By substitution :

$$P_{r,k} = \frac{d_k^2}{100}\left[\sum_{i=1}^{q}\left(\frac{B_i M_i n_{i,k}}{\sum_{k=1}^{80} d_k^2 n_{i,k}}\right) - \frac{B_s M_s n_{s,k}}{\sum_{k=1}^{80} d_k^2 n_{s,k}}\right] \tag{7}$$

The number of snippet equivalents in the kth class interval in the remainder of the delivery :

$$N_{r,k} = \frac{P_{r,k}}{\frac{\pi}{4}\rho\ell d_k^2} \tag{8}$$

The number of snippet equivalents in the remainder of the delivery :

$$N_r = \sum_{k=1}^{80} N_{r,k} = \frac{1}{\frac{\pi}{4}\rho\ell}\sum_{k=1}^{80}\frac{P_{r,k}}{d_k^2} \tag{9}$$

The number of snippets measured for the delivery :

$$n = \sum_{k=1}^{80}\sum_{i=1}^{q} n_{i,k} \tag{10}$$

Deduct the snippets measured for the component removed from the delivery:

$$n_r = \sum_{k=1}^{80}\sum_{i=1}^{q} n_{i,k} - \sum_{k=1}^{80} n_{s.k} \tag{11}$$

Assume that a random specimen of n snippets will contain a number of snippets in each class interval in the same proportions as the total number of snippet equivalents in each class interval of the remainder of the delivery and scale down to calculate the raw histogram.

$$n_k = N_{r,k} \frac{n_r}{N_r} \tag{12}$$

$$n_k = \frac{P_k}{d_k^2} * \frac{n_r}{\sum_{k=1}^{80} \frac{P_k}{d_k^2}} \tag{13}$$

By further substitution and simplification, for the remainder of the delivery :

$$n_k = \left[\sum_{i=1}^{q} \left(\frac{B_i M_i n_{i,k}}{\sum_{k=1}^{80} d_k^2 n_{i,k}} \right) - \frac{B_s M_s n_{s.k}}{\sum_{k=1}^{80} d_k^2 n_{s,k}} \right] * \left[\frac{\sum_{k=1}^{80} \sum_{i=1}^{q} n_{i,k} - \sum_{k=1}^{80} n_{s,k}}{\sum_{k=1}^{80} \left\{ \sum_{i=1}^{q} \left(\frac{B_i M_i n_{i,k}}{\sum_{k=1}^{80} d_k^2 n_{i,k}} \right) - \frac{B_s M_s n_{s,k}}{\sum_{k=1}^{80} d_k^2 n_{s,k}} \right\}} \right] \tag{14}$$